

DMM-I UNIT-V SPRINGS

Spring:- (Define spring. What is the purpose of mechanical springs?)

- ❖ A spring is defined as an elastic machine elements that deflects under the action of the load and returns to its original shape when the load is removed.
- ❖ A mechanical springs is an elastic member used to connect two bodies of two parts of a machine.

Functions and Applications of springs:- (Write the function of spring in machine)

S.No	Function	Applications
1	To absorb shocks and vibrations	<ul style="list-style-type: none">• Vehicle suspension springs• Railway buffer springs• Buffer springs in elevators• Vibration mounts for machinery
2	To store energy	<ul style="list-style-type: none">• Springs used in clocks, toys, Movie-cameras, circuit breakers and starters.
3	To measure force	<ul style="list-style-type: none">• Springs used in weighing balance and engine indicators
4	To control motion	<ul style="list-style-type: none">• Springs used in cam and follower mechanism, spring is used to maintain contact between two elements
5	To apply force	<ul style="list-style-type: none">• In engine valve mechanism, spring is used to return the rocker arm to its normal position when the disturbing force is removed.• The spring used in clutch provides the required force to engage the clutch.

Types of springs:- (Generally, how the springs are classified ? Indicate the different types of springs by sketches and give minimum two practical applications of each)

Following are important types of springs according to their shape:

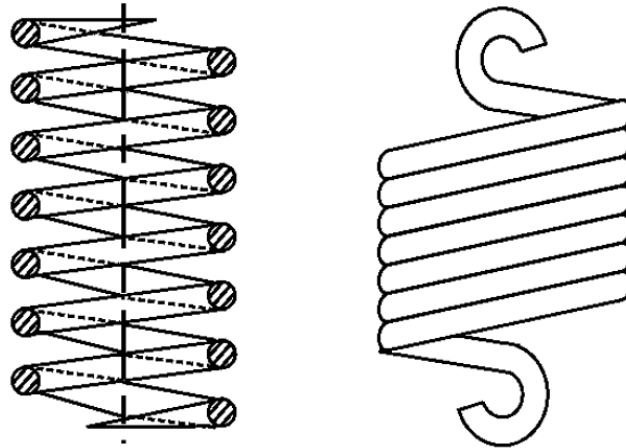
1. Helical springs.
2. Conical and volute springs.
3. Torsion springs
4. Laminated or leaf springs
5. Disc or Belleville springs.



6. Special purpose springs.

1. Helical springs:- (Compression (or) Extension springs):- The major stress is torsional shear stress due to twisting. they are made of wire coiled into helical form. The load is applied along the axis of the helix. The deflection is linear. These springs may be compression or tension springs.

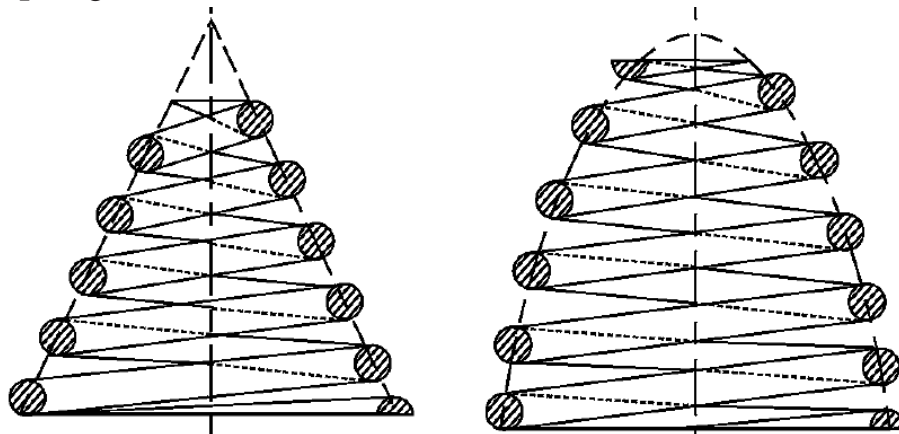
The two forms of helical springs are compression helical spring (open coiled helical springs) as shown in Fig.(a) and tension helical spring (closely coiled helical spring) as shown in Fig.(b).



(a) Compression Helical spring (b) Tension Helical Spring

2. Conical and volute springs:-

The major stresses produced in conical and volute springs are also shear stresses due to twisting. The conical spring, as shown in Fig.(a), is wound with a uniform pitch whereas the volute springs, as shown in Fig.(b), are wound in the form of paraboloid with constant pitch and lead angles. The special applications where a telescoping spring.



(a) Conical Spring

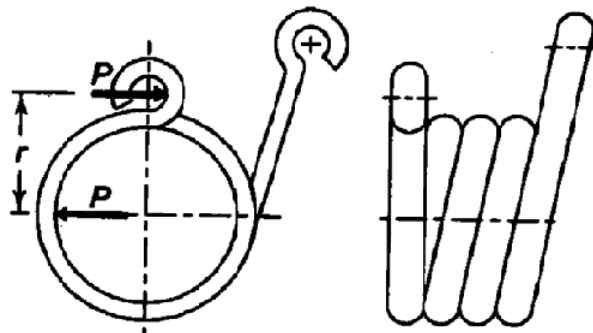
(b) Volute Spring

3. Torsion springs:- These springs may be of helical or spiral type as shown in Fig.

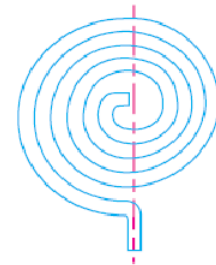
Helical torsion springs:-The major stresses are tensile and compressive due to bending. The torque is being applied about the axis of the helix. The deflection is circular.

Spiral torsion springs:- The major stresses are tensile and compressive due to bending. They consist of flat strip wound in the form of a spiral and loaded in torsion. The deflection is angular.

Torsion springs are used for electrical mechanisms, watches and clocks.



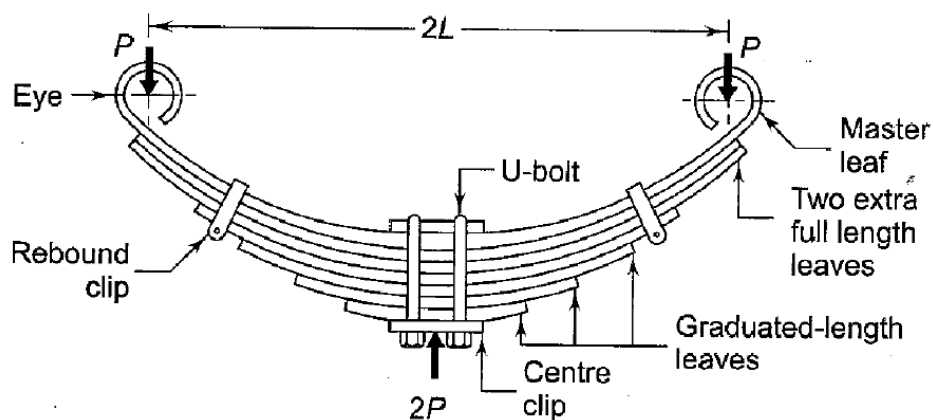
Helical torsion spring



Spiral torsion spring

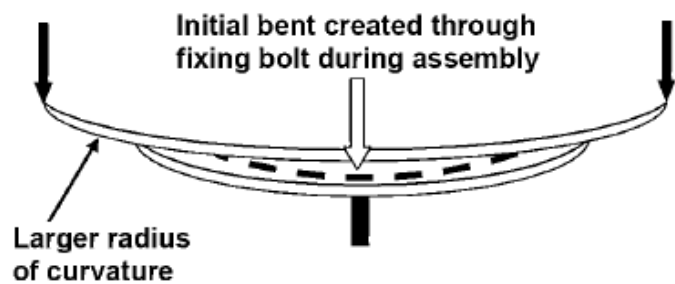
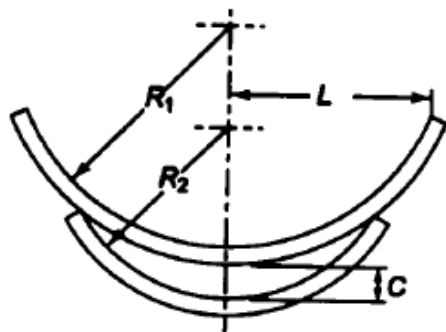
4. Laminated or leaf springs:- The laminated or leaf spring (also known as flat spring or carriage spring) consists of a number of flat plates (known as leaves) of varying lengths held together by means of clamps and bolts, as shown in Fig. These are mostly used in automobiles.

The major stresses produced in leaf springs are tensile and compressive stresses.



Laminated or leaf spring

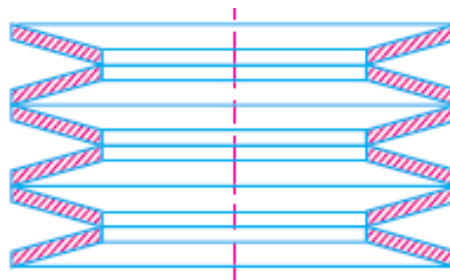
Nipping of leaf Springs:- The initial gap C between the extra full-length leaf and the graduated-length leaf before the assembly is called a nip. Such pre-stressing, achieved by a difference in radius of curvature, is known as nipping.



Nipping of leaf Springs

5. Disc or Belleville springs:- These springs consist of a number of conical discs held together against slipping by a central bolt or tube as shown in Fig. These springs are used in applications where high spring rates and compact spring units are required.

The major stresses produced in disc or Belleville springs are tensile and compressive stresses.



Disc or Belleville spring

6. Special purpose springs:- These springs are air or liquid springs, rubber springs, ring springs etc. The fluids (air or liquid) can behave as a compression spring. These springs are used for special types of application only.

Material for Helical Springs:-

The material of the spring should have high fatigue strength, high ductility, high resilience and creep resistant. Type of material largely depends upon the service of applications i.e. severe service, average service or light service.

There are four basic varieties of steel wire which are used in springs.

1. steel wire unalloyed cold drawn:

There are four Grades:	Gr.1	Static load
	Gr.2	Moderate load
	Gr.3	Moderate dynamic load
	Gr.4	Load subjected severe stresses

2. Oil -hardened and tempered spring steel wire (unalloyed)

There are two Grades:	Gr.1	General purpose
	Gr.2	Intended for valve springs subjected to high dynamic stresses

3. Oil-hardened and tempered steel wire (alloyed)
 - Used for elevated temperature
 - Static loads 2S
 - Dynamic loads 2D
4. Stainless steel spring wire for normal corrosion resistance

The springs are mostly made from oil-tempered carbon steel wires containing 0.60 to 0.70 percent carbon and 0.60 to 1.0 per cent manganese.

Music wire is used for small springs. Non-ferrous materials like phosphor bronze, beryllium copper, Monel metal, brass etc., may be used in special cases to increase fatigue resistance, temperature resistance and corrosion resistance.

Terms used in Compression Springs (Helical springs)

1. Solid length: Length of completely compressed spring $L_S = n'.d$

Where n' = Total number of coils, and

d = Diameter of the wire.

2. Free length: Overall length of compression coil spring with no application of load.

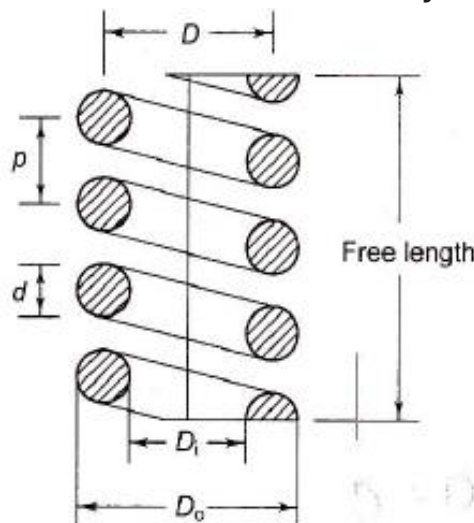
Free length of the spring, L_f = Solid length + Maximum compression + Clearance between adjacent coils (or clash allowance)

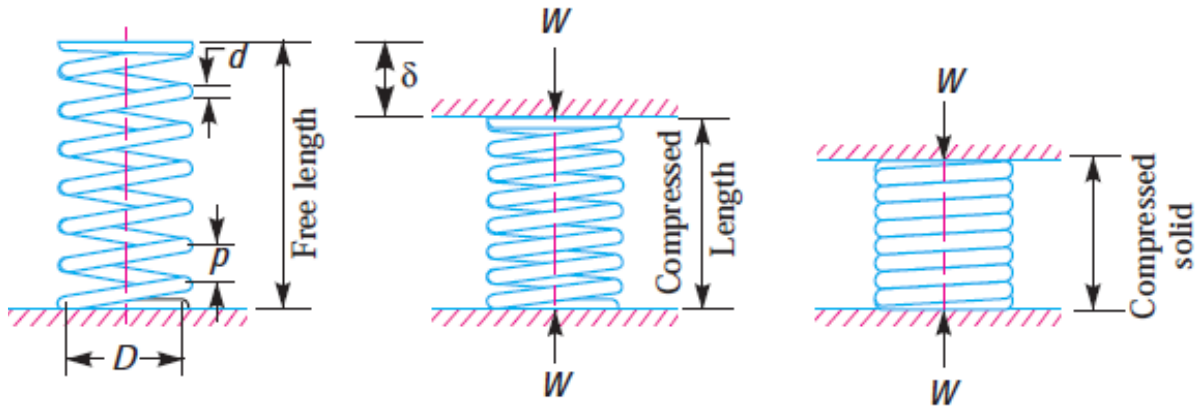
$$= n'.d + \delta_{max} + 0.15 \delta_{max}$$

The following relation may also be used to find the free length of the spring, i.e.

$$L_F = n'.d + \delta_{max} + (n' - 1) \times 1 \text{ mm}$$

In this expression, the clearance between the two adjacent coils is taken as 1 mm.





Compression Spring Nomenclature

3. Spring index: The spring index is defined as the ratio of the mean diameter to wire diameter

$$\text{Spring index, } C = D / d$$

where D = Mean diameter of the coil, and

d = Diameter of the wire.

4. Stiffness (or) Spring rate (or) Spring constant (k): The spring rate is defined as the load required per unit deflection of the spring.

$$k = W / \delta$$

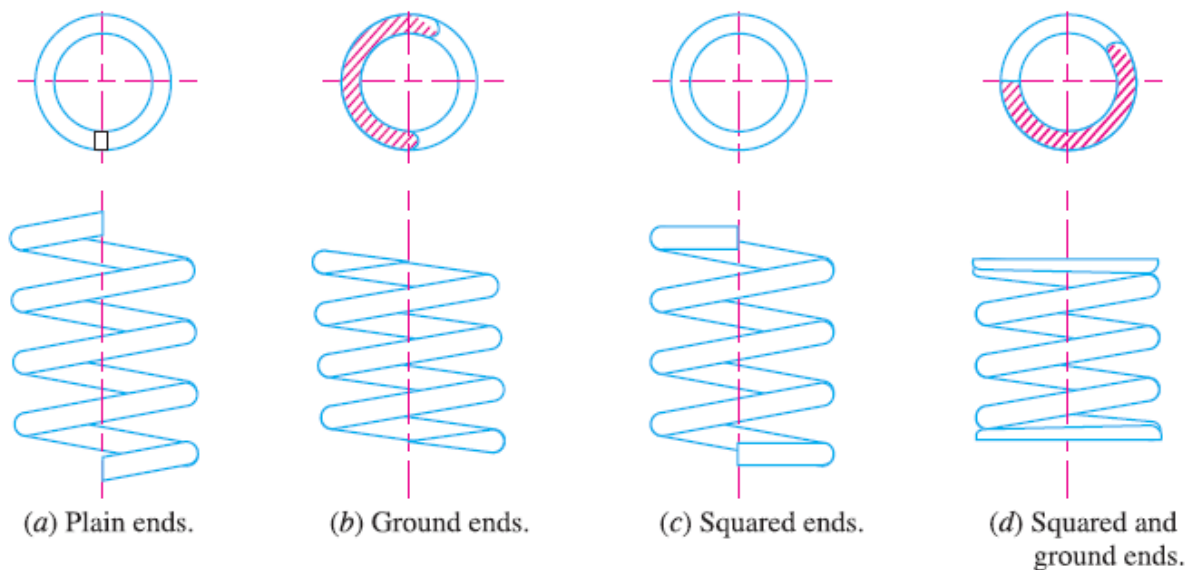
W = Load, and

δ = Deflection of the spring.

5. Pitch. The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state.

$$\text{Pitch of the coil, } p = \frac{\text{Free length}}{n' - 1}$$

End Connections for Compression Helical Springs:- a). Plain ends, b). Ground ends c). Squared ends, d). Squared and ground ends.



Stresses and deflection of helical compression springs subjected to

Axial Loading:- (Explain the design of helical compression springs with a neat sketch (or) Discuss the stresses in Helical springs of circular wire)

D = Mean diameter of the spring coil,

d = Diameter of the spring wire,

n = Number of active coils,

G = Modulus of rigidity for the spring material,

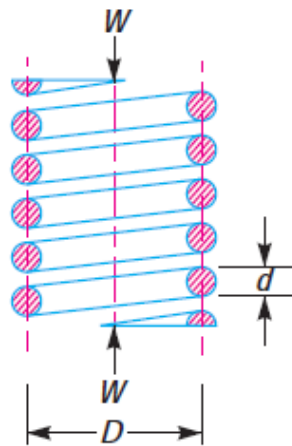
W = Axial load on the spring,

τ = Maximum shear stress induced in the wire,

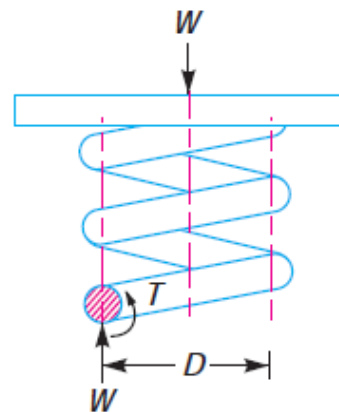
C = Spring index = D/d ,

p = Pitch of the coils, and

δ = Deflection of the spring, as a result of an axial load W .



(a) Axially loaded helical spring.



(b) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

The wire is subjected to torsion(T), and direct load (W). Shear stress are setup within the material of the wire as follows.

1. Shear stress due to axial force (direct shear stress)
2. Shear stress due to twisting moment.

Shear stress due to axial force (or) direct shear stress,

$$\begin{aligned} \tau_d &= \frac{\text{Load}}{\text{Cross-sectional area of the wire}} \\ &= \frac{W}{\frac{\pi}{4} \times d^2} = \frac{4W}{\pi d^2} \end{aligned}$$

Shear stress due to twisting moment

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_t \times d^3$$

$$\tau_t = \frac{8W.D}{\pi d^3}$$

$$\text{Total shear stress } \tau = \tau_d \pm \tau_t = \frac{4W}{\pi d^2} \pm \frac{8W.D}{\pi d^3}$$

The **positive** sign is used for the inner edge of the wire and **negative** sign is used for the outer edge of the wire. Since the stress is maximum at the inner edge of the wire,

$$= \frac{4W}{\pi d^2} + \frac{8W.D}{\pi d^3}$$

$$= \frac{8W.D}{\pi d^3} + \frac{4W}{\pi d^2} = \frac{8W.D}{\pi d^3} \left(1 + \frac{d}{2D}\right)$$

$$= \frac{8W.D}{\pi d^3} \left(1 + \frac{1}{2C}\right) = K_s \times \frac{8W.D}{\pi d^3} \quad \dots \text{(Substituting } D/d = C)$$

where $K_s = \text{Shear stress factor} = 1 + \frac{1}{2C}$

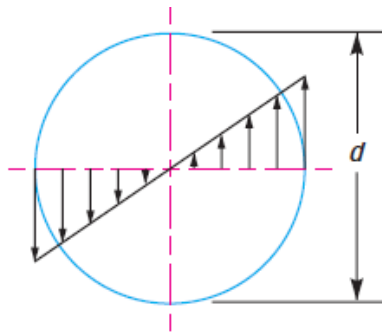
In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl's stress factor (K) introduced by A.M. Wahl may be used.

\therefore Maximum shear stress induced in the wire,

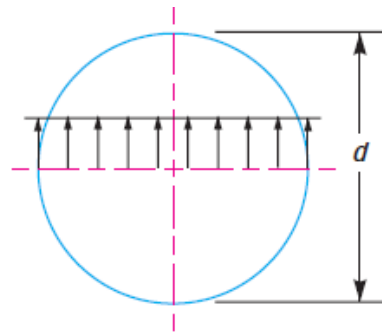
$$\tau = K \times \frac{8W.D}{\pi d^3} \quad \dots \text{(Substituting } D/d = C)$$

$$= K \times \frac{8W.C}{\pi d^2}$$

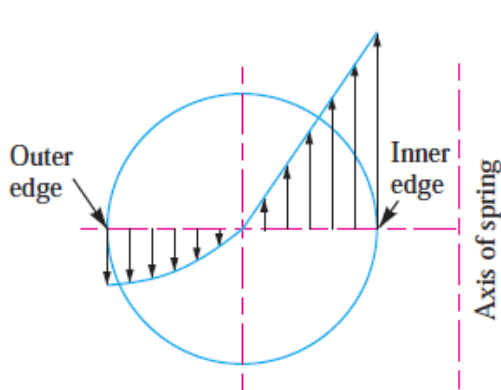
where $K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$



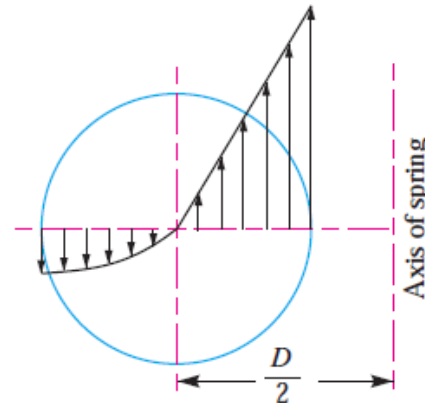
(a) Torsional shear stress diagram.



(b) Direct shear stress diagram.



(c) Resultant torsional shear and direct shear stress diagram.



(d) Resultant torsional shear, direct shear and curvature shear stress diagram.

Note: The Wahl's stress factor (K) may be considered as composed of two sub-factors, K_S and K_C , such that

$$K = K_S \times K_C$$

where

K_S = Stress factor due to shear, and

K_C = Stress concentration factor due to curvature.

Deflection of Helical Springs of Circular Wire:- (Write the energy storage capacity of springs)

In the previous article, we have discussed the maximum shear stress developed in the wire. We know that

Total active length of the wire,

$$l = \text{Length of one coil} \times \text{No. of active coils} = \pi D \times n$$

Let

θ = Angular deflection of the wire when acted upon by the torque T .

\therefore Axial deflection of the spring,

$$\delta = \theta \times D/2 \quad \dots\dots(i)$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G\theta}{l}$$

$$\theta = \frac{Tl}{J.G} \quad \dots \left(\text{considering } \frac{T}{J} = \frac{G\theta}{l} \right)$$

where

J = Polar moment of inertia of the spring wire

$$= \frac{\pi}{32} \times d^4, \quad d \text{ being the diameter of spring wire.}$$

and

G = Modulus of rigidity for the material of the spring wire.

$$\theta = \frac{Tl}{J.G} = \frac{\left(W \times \frac{D}{2} \right) \pi D.n}{\frac{\pi}{32} \times d^4 G} = \frac{16W.D^2.n}{G.d^4} \quad \dots \text{(ii)}$$

Substituting this value of θ in equation (i), we have

$$\delta = \frac{16W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8W.D^3.n}{G.d^4} = \frac{8W.C^3.n}{G.d} \quad \dots (\because C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{G.d^4}{8D^3.n} = \frac{G.d}{8C^3.n} = \text{constant}$$

Assuming that the load is applied gradually, the energy stored in a spring is,

$$U = \frac{1}{2} W \delta$$

Note: When a load (say P) falls on a spring through a height h , then the energy absorbed in a spring is given by

$$U = P(h + \delta) = \frac{1}{2} W.\delta$$

W = Equivalent static load *i.e.* the gradually applied load which shall produce the same effect as by the falling load P , and

δ = Deflection produced in the spring.

Buckling of Compression Springs:-

It has been found experimentally that when the free length of the spring (L_F) is more than four times the mean or pitch diameter (D), then the spring behaves like a column and may fail by buckling at a comparatively low load as shown in Fig.

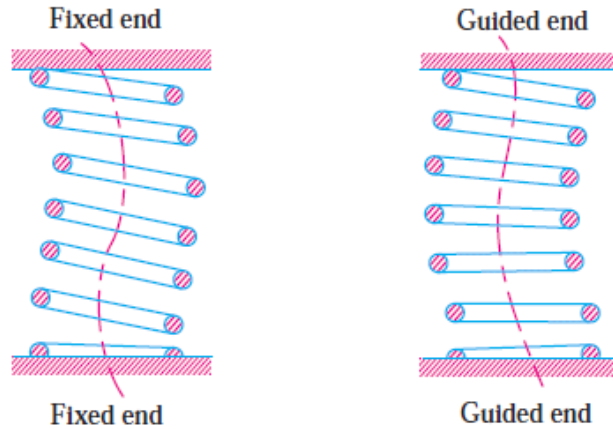


Fig: Buckling of compression spring

The critical axial load (W_{cr}) that causes buckling may be calculated by using the following relation, i.e.

$$W_{cr} = k \times K_B \times L_F$$

k = Spring rate or stiffness of the spring = W/δ ,

L_F = Free length of the spring, and

K_B = Buckling factor depending upon the ratio L_F / D .

Surging (Spring surge) and Critical frequency:-

If a compression spring is held at one end and the other end is deflected by a suddenly applied load, the coil of the spring will not instantaneously have the same deflection. End coil in contact with the load deflects first and transmits a large part of its deflection to the next coil. Thus the deflection wave travels from one end to the other where it gets reflected and travels back. This travelling back and forth of the deflection wave, i.e. vibration with natural frequency dies out because of damping. If the frequency of the applied force equals the natural frequency of the spring, resonance known as surging occurs with large deflections of the coils and the spring may fail due to high stresses.

The natural frequency of spring should be at least 12 times the frequency of applied force to avoid resonance with all harmonic frequencies.

$$f \geq 12 f_i$$

f_i = Frequency of the applied force,

when $f = f_i$ then, it is called **critical frequency**

Surging is a problem in valve springs in I.C Engines

The natural frequency for springs clamped between two plates is given by

$$f_n = \frac{d}{2\pi D^2 n} \sqrt{\frac{6 G \cdot g}{\rho}} \text{ cycles/s}$$

d = Diameter of the wire,
 D = Mean diameter of the spring,
 n = Number of active turns,
 G = Modulus of rigidity,
 g = Acceleration due to gravity, and
 ρ = Density of the material of the spring.

How does surge in springs eliminated:-

The surge in springs may be eliminated by using the following methods :

1. By using friction dampers on the centre coils so that the wave propagation dies out.
2. By using springs of high natural frequency.
3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

Problem (1):- A compression coil spring made of an alloy steel is having the following specifications : Mean diameter of coil = 50 mm ; Wire diameter = 5 mm ; Number of active coils = 20. If this spring is subjected to an axial load of 500 N ; calculate the maximum shear stress (neglect the curvature effect) to which the spring material is subjected.

Given : $D = 50$ mm ; $d = 5$ mm ; $n = 20$; $W = 500$ N

We know that the spring index,

$$C = \frac{D}{d} = \frac{50}{5} = 10$$

∴ Shear stress factor,

$$K_s = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 10} = 1.05$$

and maximum shear stress (neglecting the effect of wire curvature),

$$\begin{aligned} \tau &= K_s \times \frac{8W \cdot D}{\pi d^3} = 1.05 \times \frac{8 \times 500 \times 50}{\pi \times 5^3} = 534.7 \text{ N/mm}^2 \\ &= 534.7 \text{ MPa} \end{aligned}$$

Problem (2):- A helical spring is made from a wire of 6 mm diameter and has outside diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 kN/mm², find the axial load which the spring can carry and the deflection per active turn.

Given : $d = 6$ mm ; $D_o = 75$ mm ; $\tau = 350$ MPa = 350 N/mm² ; $G = 84$ kN/mm²
 $= 84 \times 10^3$ N/mm²

mean diameter of the spring,

$$D = D_o - d = 75 - 6 = 69 \text{ mm}$$

$$\therefore \text{Spring index, } C = \frac{D}{d} = \frac{69}{6} = 11.5$$

Let $W =$ Axial load, and
 $\delta / n =$ Deflection per active turn.

1. Neglecting the effect of curvature

We know that the shear stress factor,

$$K_s = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 11.5} = 1.043$$

and maximum shear stress induced in the wire (τ),

$$350 = K_s \times \frac{8 W \cdot D}{\pi d^3} = 1.043 \times \frac{8 W \times 69}{\pi \times 6^3} = 0.848 W$$

$$W = 350 / 0.848 = 412.7 \text{ N}$$

We know that deflection of the spring,

$$\delta = \frac{8 W \cdot D^3 \cdot n}{G \cdot d^4}$$

\therefore Deflection per active turn,

$$\frac{\delta}{n} = \frac{8 W \cdot D^3}{G \cdot d^4} = \frac{8 \times 412.7 (69)^3}{84 \times 10^3 \times 6^4} = 9.96 \text{ mm}$$

2. Considering the effect of curvature

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.123$$

maximum shear stress induced in the wire (τ),

$$350 = K \times \frac{8 W \cdot C}{\pi d^2} = 1.123 \times \frac{8 \times W \times 11.5}{\pi \times 6^2} = 0.913 W$$

$$W = 350 / 0.913 = 383.4 \text{ N}$$

and deflection of the spring,

$$\delta = \frac{8 W \cdot D^3 \cdot n}{G \cdot d^4}$$

\therefore Deflection per active turn,

$$\frac{\delta}{n} = \frac{8 W \cdot D^3}{G \cdot d^4} = \frac{8 \times 383.4 (69)^3}{84 \times 10^3 \times 6^4} = 9.26 \text{ mm}$$

Problem (3):-A closely coiled helical spring is made of 10 mm diameter steel wire, the coil consisting of 10 complete turns with a mean diameter of 120 mm. The spring carries an axial pull of 200 N. Determine the shear stress induced in the spring neglecting the effect of stress concentration. Determine also the deflection in the spring, its stiffness and strain energy stored by it if the modulus of rigidity of the material is 80 kN/mm².

Given : $d = 10 \text{ mm}$; $n = 10$; $D = 120 \text{ mm}$; $W = 200 \text{ N}$; $G = 80 \text{ kN/mm}^2 = 80 \times 10^3 \text{ N/mm}^2$

Shear stress induced in the spring neglecting the effect of stress concentration

We know that shear stress induced in the spring neglecting the effect of stress concentration is,

$$\tau = \frac{8 W . D}{\pi d^3} \left(1 + \frac{d}{2D} \right) = \frac{8 \times 200 \times 120}{\pi (10)^3} \left[1 + \frac{10}{2 \times 120} \right] \text{ N/mm}^2$$

$$= 61.1 \times 1.04 = 63.54 \text{ N/mm}^2 = 63.54 \text{ MPa}$$

Deflection in the spring

We know that deflection in the spring,

$$\delta = \frac{8 W . D^3 n}{G . d^4} = \frac{8 \times 200 (120)^3 10}{80 \times 10^3 (10)^4} = 34.56 \text{ mm}$$

Stiffness of the spring

We know that stiffness of the spring

$$= \frac{W}{\delta} = \frac{200}{34.56} = 5.8 \text{ N/mm}$$

Strain energy stored in the spring

We know that strain energy stored in the spring,

$$U = \frac{1}{2} W . \delta = \frac{1}{2} \times 200 \times 34.56 = 3456 \text{ N-mm} = 3.456 \text{ N-m}$$

Problem (4):-Derive the expressions for Springs in Series and Springs in Parallel:-

Springs in Series

Consider two springs connected in series as shown in Fig.

Let $W =$ Load carried by the springs,

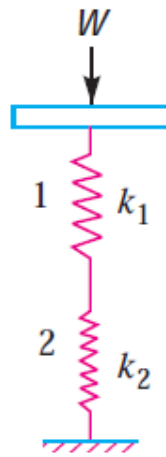
$\delta_1 =$ Deflection of spring 1,

$\delta_2 =$ Deflection of spring 2,

$k_1 =$ Stiffness of spring 1 = W / δ_1 , and

$k_2 =$ Stiffness of spring 2 = W / δ_2





Springs in series

A little consideration will show that when the springs are connected in series, then the total deflection produced by the springs is equal to the sum of the deflections of the individual springs.

∴ Total deflection of the springs,

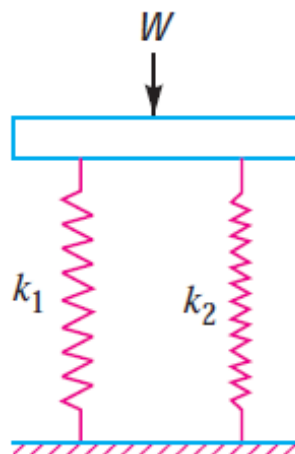
$$\delta = \delta_1 + \delta_2$$

$$\frac{W}{k} = \frac{W}{k_1} + \frac{W}{k_2}$$

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

k = Combined stiffness of the springs.

Springs in Parallel



Springs in Parallel

Consider two springs connected in parallel as shown in Fig

Let W = Load carried by the springs,

W_1 = Load shared by spring 1,

W_2 = Load shared by spring 2,

k_1 = Stiffness of spring 1, and

k_2 = Stiffness of spring 2.

A little consideration will show that when the springs are connected in parallel, then the total deflection produced by the springs is same as the deflection of the individual springs.

$$W = W_1 + W_2$$

$$\delta.k = \delta.k_1 + \delta.k_2$$

$$k = k_1 + k_2$$

where k = Combined stiffness of the springs, and
 δ = Deflection produced.

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